$\boxed{\text{Função Inversa. Função Implícita}}$

Inversa em $\mathbb{R}$: Exemplo:

$$f(x) = \operatorname{sen} x \qquad y = \operatorname{sen} x$$

$$\iff x = \operatorname{arcsen} y$$

$$f^{-1}(x) = \operatorname{arcsen}(x)$$

$$f^{-1}(y) = \operatorname{arcsen}(y)$$

$$f'(x) = \cos x; \quad \left(f^{-1}\right)'(y) = \frac{1}{f'(\operatorname{arcsen} y)}$$

$$\operatorname{arcsen}(y)' = \frac{1}{\cos(\operatorname{arcsen} y)} = \frac{1}{\sqrt{1-\operatorname{sen}^2(\operatorname{arcsen} y)}}$$

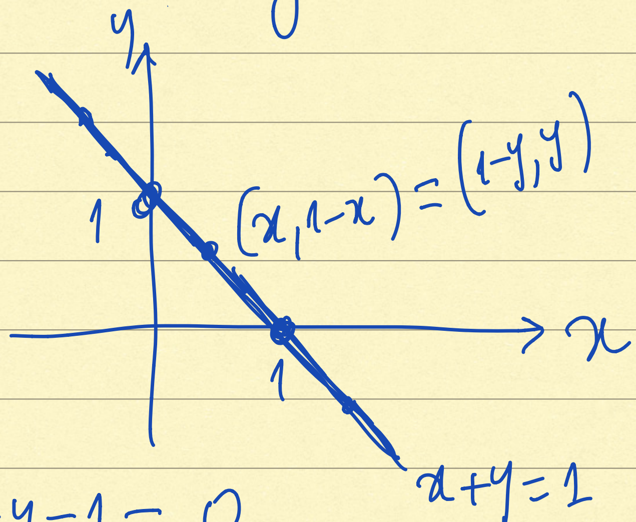
$$= \frac{1}{\sqrt{1-y^2}}$$

$$\begin{tikzcd} x \arrow[r, bend left, "f"] & y \arrow[l, bend left, "f^{-1}"] \end{tikzcd}$$

$\mathbb{R}^n$???

Exemplos: Implicita

1) $x + y = 1$ em $\mathbb{R}^2$



$$x + y = 1 \iff y = \underbrace{1-x} \iff x = \underbrace{1-y}$$

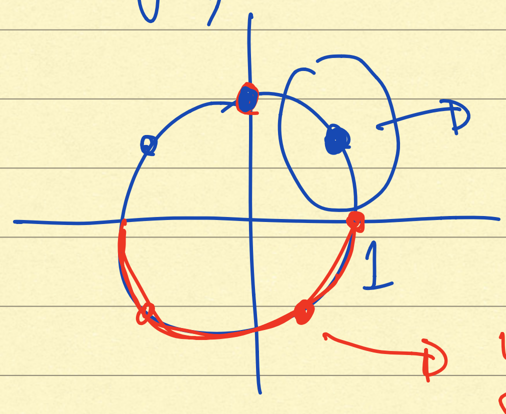
$$x + y - 1 = 0$$

$$F(x,y) = 0 \qquad F: \mathbb{R}^2 \to \mathbb{R}$$
$$F(x,y) = x + y - 1$$

$$\Updownarrow$$

$$\left.\begin{array}{l} y = f(x) = 1 - x \\ \Updownarrow \\ x = h(y) = 1 - y \end{array}\right\} \text{Sabendo fazer as contas!}$$

2) $x^2+y^2=1$ em $\mathbb{R}^2$



$y=\sqrt{1-x^2},\ y>0$

$y=-\sqrt{1-x^2},\ y<0$

Contas:

$F(x,y)=0 \Longleftrightarrow y=f(x)$

$x^2+y^2-1=0$

localmente

$f \equiv$ função implícita

"escondida"

(A equação $F(x,y)=0$ "esconde" a função $f(x)=y$)

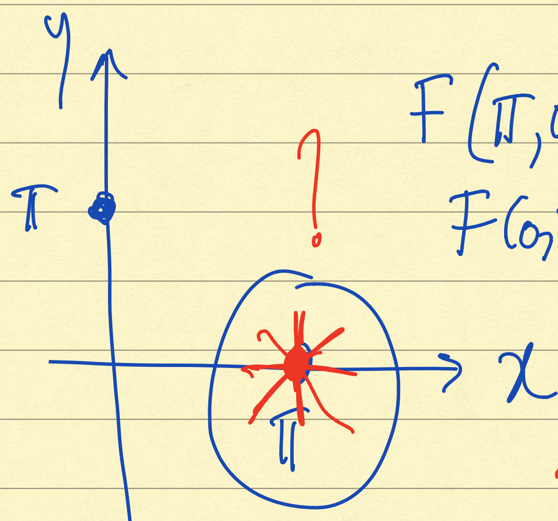
3) $\operatorname{sen}(x+y) + xy = 0$, $\mathbb{R}^2$

$F(x,y) = 0$ $\quad F: \mathbb{R}^2 \to \mathbb{R}$.

Não é possível exprimir y como função de x ou x como função de y.

$F(\pi,0) = 0$

$F(0,\pi) = 0$



~~contas~~

Será que $F(x,y) = 0$ ~~contas~~ $\Leftrightarrow$

$y = f(x)$

"perto" do ponto $(\pi, 0)$?

$\boxed{\text{localmente}}$

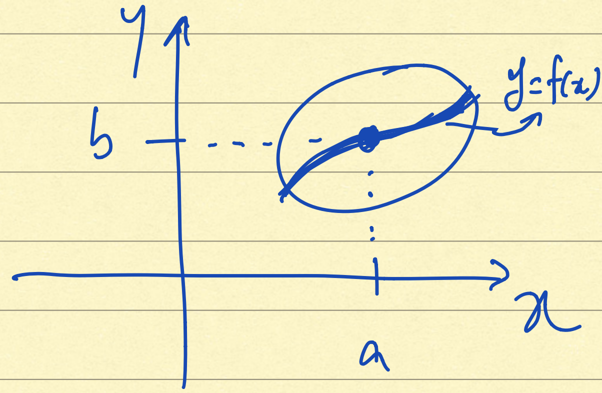
desconhecida!

Será que existe?

Suponhamos que $F(x,y)=0$

$F:\mathbb{R}^2 \to \mathbb{R}$, C^1. $F(a,b)=0$

localmente em (a,b) existe $f:\mathbb{R}\to\mathbb{R}$ tal que



$F(x,y)=0 \iff y=f(x)$.

Então

$$F(x,y)=0$$

$$F(x,f(x))=0 \quad \text{Composta!}$$

Derivando, obtem-se:

$$\frac{\partial F}{\partial x}(x,f(x)) + \frac{\partial F}{\partial y}(x,f(x))\,f'(x)=0$$

em (a,b) , $b = f(a)$

$F(a,b) = 0$

$F(a, f(a)) = 0$

$$\frac{\partial F}{\partial x}(a,b) + \underbrace{\boxed{\frac{\partial F}{\partial y}(a,b)}}_{\neq 0} f'(a) = 0$$

Se $\frac{\partial F}{\partial y}(a,b) \neq 0$, então

$$\boxed{f'(a) = - \frac{\frac{\partial F}{\partial x}(a,b)}{\frac{\partial F}{\partial y}(a,b)}}$$

derivada da implícita!!!

$x + y = 1$ $\qquad\qquad$ $ax + by = c$

$\Leftrightarrow y = \frac{c}{b} - \frac{a}{b}x$

$\boxed{b \neq 0}$

$F(x,y) = x + y - 1 = 0$

$$DF(x,y) = \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} \frac{\partial F}{\partial x} & \boxed{\frac{\partial F}{\partial y}} \end{bmatrix}$$

$b \neq 0$ $\qquad\qquad$ $\frac{\partial F}{\partial y} \neq 0$

— $\shortparallel$ ——— ?

Truque: $F(x,y) = 0 \Leftrightarrow y = f(x)$

y variável dependente de x.

x variável livre!

$$\{(x,y)\in\mathbb{R}^2 : F(x,y)=0\} =$$

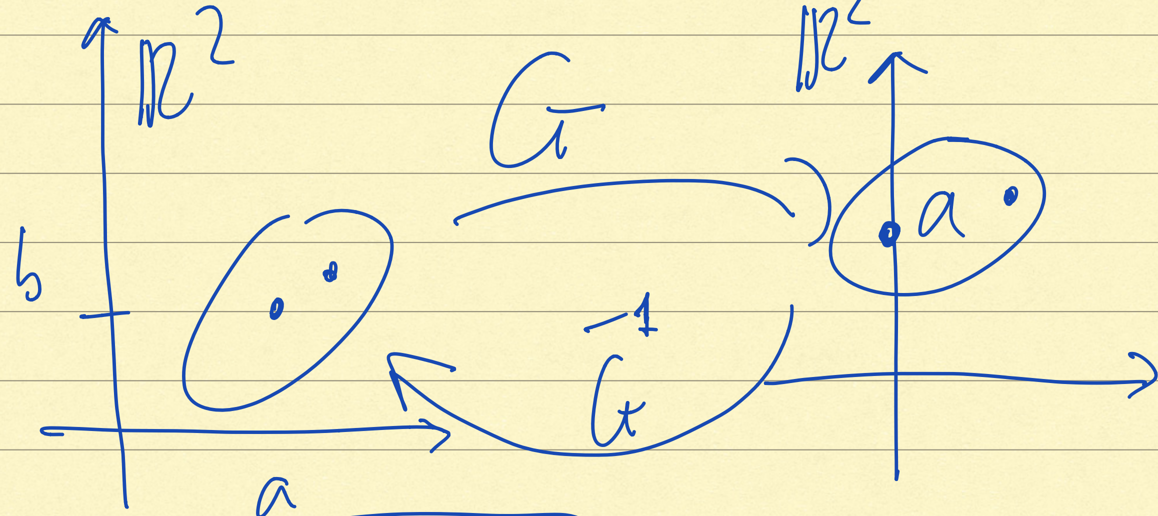
$$\{(x,y)\in\mathbb{R}^2 : F(x,y)=0 \; ; \; x=x\}$$

$$\begin{cases} F(x,y)=0 \\ x=x \end{cases}$$

$$G(x,y) = \big(F(x,y),\, x\big) = (0,x)$$

$$G : \mathbb{R}^2 \longrightarrow \mathbb{R}^2, \quad C^1$$

$$G(a,b) = \big(F(a,b),\, a\big) = (0,a)$$



Se $\boxed{G^{-1} \text{ existir}}$, então

$$G(x,y) = (0,x)$$

$$\Leftrightarrow \underbrace{(x,y)} = \underbrace{G^{-1}(0,x)}_{\text{só depende de } x}$$

existe f tal que:

$$y = f(x)$$

$\boxed{\text{implícita}}$

Falta então ver qual a condição que garante a existência de G^{-1}.